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**Mathematics**  
**Higher level**  
**Paper 2**

Tuesday 19 November 2019 (morning)

Candidate session number

2 hours

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**Instructions to candidates**

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics HL and further mathematics HL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[100 marks]**.





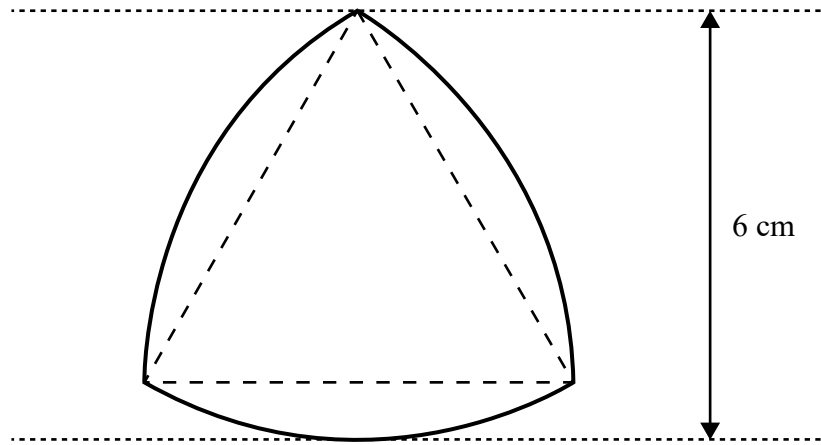




4. [Maximum mark: 7]

The following shape consists of three arcs of a circle, each with centre at the opposite vertex of an equilateral triangle as shown in the diagram.

diagram not to scale



For this shape, calculate

(a) the perimeter;

[2]

(b) the area.

[5]

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7. [Maximum mark: 6]

Runners in an athletics club have season's best times for the 100 m, which can be modelled by a normal distribution with mean 11.6 seconds and standard deviation 0.8 seconds. To qualify for a particular competition a runner must have a season's best time of under 11 seconds. A runner from this club who has qualified for the competition is selected at random. Find the probability that he has a season's best time of under 10.7 seconds.

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8. [Maximum mark: 8]

Eight boys and two girls sit on a bench. Determine the number of possible arrangements, given that

- (a) the girls do not sit together; [3]
- (b) the girls do not sit on either end; [2]
- (c) the girls do not sit on either end and do not sit together. [3]

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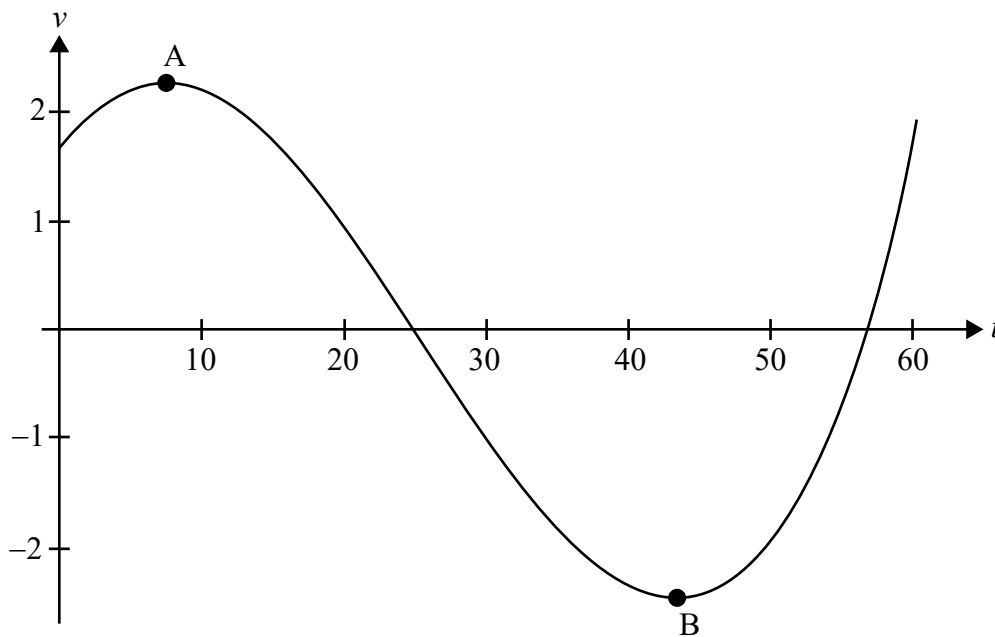
### Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

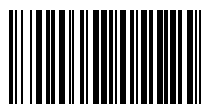
9. [Maximum mark: 14]

A body moves in a straight line such that its velocity,  $v \text{ m s}^{-1}$ , after  $t$  seconds is given by  $v = 2 \sin\left(\frac{t}{10} + \frac{\pi}{5}\right) \csc\left(\frac{t}{30} + \frac{\pi}{4}\right)$  for  $0 \leq t \leq 60$ .

The following diagram shows the graph of  $v$  against  $t$ . Point A is a local maximum and point B is a local minimum.



- (a) (i) Determine the coordinates of point A and the coordinates of point B.
- (ii) Hence, write down the maximum speed of the body. [5]
- (b) The body first comes to rest at time  $t = t_1$ . Find
  - (i) the value of  $t_1$ ;
  - (ii) the distance travelled between  $t = 0$  and  $t = t_1$ ;
  - (iii) the acceleration when  $t = t_1$ . [6]
- (c) Find the distance travelled in the first 30 seconds. [3]



Do **not** write solutions on this page.

10. [Maximum mark: 19]

A random variable  $X$  has probability density function

$$f(x) = \begin{cases} 3a & , 0 \leq x < 2 \\ a(x-5)(1-x) & , 2 \leq x \leq b \\ 0 & , \text{otherwise} \end{cases} \quad a, b \in \mathbb{R}^+, 3 < b \leq 5.$$

(a) Find, in terms of  $a$ , the probability that  $X$  lies between 1 and 3. [4]

Consider the case where  $b = 5$ .

(b) Sketch the graph of  $f$ . State the coordinates of the end points and any local maximum or minimum points, giving your answers in terms of  $a$ . [4]

(c) Find the value of

(i)  $a$ ;

(ii)  $E(X)$ ;

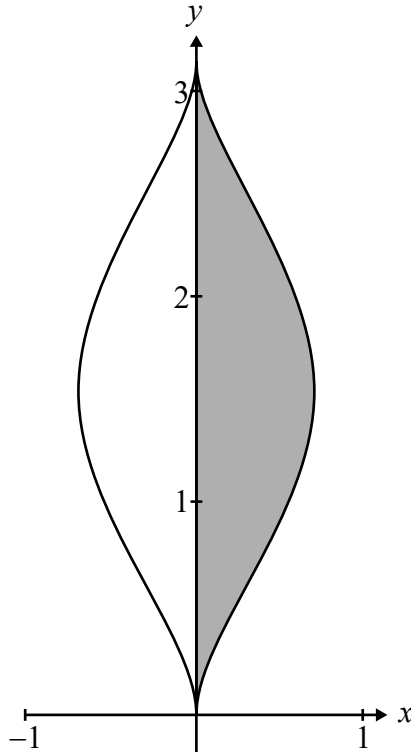
(iii) the median of  $X$ . [11]



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11. [Maximum mark: 17]

The following diagram shows part of the graph of  $2x^2 = \sin^3 y$  for  $0 \leq y \leq \pi$ .



(a) (i) Using implicit differentiation, find an expression for  $\frac{dy}{dx}$ .

(ii) Find the equation of the tangent to the curve at the point  $\left(\frac{1}{4}, \frac{5\pi}{6}\right)$ . [8]

The shaded region  $R$  is the area bounded by the curve, the  $y$ -axis and the lines  $y = 0$  and  $y = \pi$ .

(b) Find the area of  $R$ . [3]

The region  $R$  is now rotated about the  $y$ -axis, through  $2\pi$  radians, to form a solid.

(c) By writing  $\sin^3 y$  as  $(1 - \cos^2 y) \sin y$ , show that the volume of the solid formed is  $\frac{2\pi}{3}$ . [6]

